UdeM-GPP-TH-00-77

IMSc-99/05/18

Searching for New Physics via CP Violation in $B \to \pi\pi$

David London [a,1], Nita Sinha [b,2] and Rahul Sinha [b,3]

a: *Laboratoire René J.-A. Lévesque, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7*

b: *Institute of Mathematical Sciences, Taramani, Chennai 600113, India*

(February 1, 2008)

Abstract

It is well known that one can use $B \to \pi\pi$ decays to probe the CP-violating phase α. In this paper we show that these same decays can be used to search for new physics. This is done by comparing two weak phases which are equal in the standard model: the phase of the t-quark contribution to the $b \to d$ penguin amplitude, and the phase of B_d^0–$\overline{B_d^0}$ mixing. In order to make such a comparison, we require one piece of theoretical input, which we take to be a prediction for $|P/T|$, the relative size of the penguin and tree contributions to $B_d^0 \to \pi^+\pi^-$. If independent knowledge of α is available, the decay $B_d^0(t) \to \pi^+\pi^-$ alone can be used to search for new physics. If a full isospin analysis can be done, then new physics can be found solely through measurements of $B \to \pi\pi$ decays. The most promising scenario occurs when the isospin analysis can be combined with independent knowledge of α. In all cases, the prospects for detecting new physics in $B \to \pi\pi$ decays can be greatly improved with the help of additional measurements which will remove discrete ambiguities.

[1]london@lps.umontreal.ca

[2]nita@imsc.ernet.in

[3]sinha@imsc.ernet.in

1 Introduction

Within the standard model (SM), CP violation is due to the presence of a nonzero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. This phase information can be elegantly displayed using the unitarity triangle [1], in which the interior (CP-violating) angles are called α, β and γ. In the near future, these CP angles will be extracted from the measurements of rate asymmetries in B decays [2]. As usual, the hope is that these measurements will reveal the presence of physics beyond the SM.

The canonical decay modes which will be used to measure the CP angles are $B_d^0(t) \to \pi^+\pi^-$ (α), $B_d^0(t) \to J/\Psi K_S$ (β) and $B^\pm \to DK^\pm$ (γ) [3]. Assuming that each decay is dominated by a single amplitude, the corresponding CP angle can be extracted with no hadronic uncertainties. Unfortunately, this assumption does not hold for the decay $B_d^0 \to \pi^+\pi^-$: in addition to the tree contribution T, there is also a penguin contribution P which may be sizeable [4]. Nevertheless, it is still possible to obtain α without hadronic uncertainties if a $B \to \pi\pi$ isospin analysis can be performed [5].

If new physics is present, it will contribute principally at loop-level, affecting B_q^0–$\overline{B_q^0}$ mixing ($q = d, s$) [6] and/or the $b \to q$ flavour-changing neutral current (FCNC) penguin amplitudes [7]. There are a variety of ways of detecting this new physics. For example, if there is an inconsistency between the unitarity triangle as constructed from measurements of the angles and that constructed from independent measurements of the sides, this will be a signal of new physics. However, there are two potential difficulties with this. First, measurements of the sides of the triangle require theoretical input regarding certain hadronic quantities. Depending on the size of the discrepancy, one might question the precision of the theoretical numbers. Second, there are discrete ambiguities in extracting the angles, and it may be necessary to resolve these ambiguities in order to be certain that a discrepancy is in fact present [8].

A more direct way of looking for new physics is to consider two CP asymmetries which in the SM are supposed to probe the same CP angle. For example, the angle γ can be measured via $B^\pm \to DK^\pm$ [3] or $B_s^0(t) \to D_s^\pm K^\mp$ [9]. If the measured values of γ in these two modes disagree, this is a clear sign of new physics. Similarly, the angle β can be measured via $B_d^0(t) \to J/\Psi K_S$ or $B_d^0(t) \to \phi K_S$ [10]. Another example, similar in spirit to these, is the decay $B_s^0(t) \to J/\Psi\phi$.

Within the SM, the CP asymmetry in this decay is expected to vanish, so that a nonzero value would indicate the presence of new physics.

In all of these examples, the new physics affects the $b \to s$ FCNC, either through B_s^0-$\overline{B_s^0}$ mixing or the $b \to s$ penguin amplitude. One might then wonder whether it is possible to detect new physics in this way if it affects only the $b \to d$ FCNC. For example, in the Wolfenstein parametrization [11], the weak phase of B_d^0-$\overline{B_d^0}$ mixing and of the t-quark contribution to the $b \to d$ penguin are both equal to $-\beta$ in the SM. However, in the presence of new physics, these phases could be different. Therefore, if one could measure these two phases and find a discrepancy, this would be a clear signal of new physics.

Unfortunately, in a recent paper [12], we showed that this is not possible. In the SM, the largest contribution to the $b \to d$ penguin comes from an internal t-quark, and is proportional to $V_{tb}^* V_{td}$. However, the contributions of an internal u-quark ($V_{ub}^* V_{ud}$) and an internal c-quark ($V_{cb}^* V_{cd}$) are not negligible [13]. It is therefore impossible to isolate any single contribution: using the unitarity of the CKM matrix,

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \ , \tag{1}$$

it is always possible to write one amplitude in terms of the other two. And because one cannot isolate the t-quark amplitude, one cannot cleanly measure its phase. In Ref. [12], we refer to this as the "CKM ambiguity." However, we also note that the CKM ambiguity can be resolved if one makes an assumption regarding the hadronic parameters involved in the $b \to d$ FCNC amplitude.

In this paper, we apply this idea to $B_d^0(t) \to \pi^+\pi^-$. As mentioned earlier, this decay receives contributions from both a tree-level amplitude T and a $b \to d$ penguin amplitude P. The isospin analysis essentially allows one to remove this penguin "pollution" and hence obtain a clean measurement of α. Of course, as argued above, there is not enough information to extract the phase of the t-quark contribution to the $b \to d$ penguin. However, if we make an assumption about the relative size of P and T, this provides us with the additional piece of information necessary to test for the presence of new physics. As we will show, in principle this method does indeed work: given an assumption about $|P/T|$, the isospin analysis can be used not only to obtain α cleanly, but also to see if new physics is present.

In fact, the isospin analysis is not even necessary. In the absence of new physics, the ratio $|P/T|$ depends only on α and the quantities measured in $B_d^0(t) \to \pi^+\pi^-$ [14]. Thus, if independent information about α is available, the measurement of $B_d^0(t) \to \pi^+\pi^-$ alone will suffice to obtain $|P/T|$. If new physics is present, and affects the magnitude of P, then obviously the extracted value of $|P/T|$ will differ from its SM value. However, a more interesting scenario is if the only effect of new physics is to produce a discrepancy between the weak phase of B_d^0-$\overline{B_d^0}$ mixing and that of the t-quark contribution to the $b \to d$ penguin. What is perhaps not obvious, but is in fact true, as we will show, is that even in this case, the extracted value of $|P/T|$ will still differ from that which one would have obtained in the absence of new physics. Therefore, given a prediction for $|P/T|$ and some knowledge of α (either from independent measurements or via an isospin analysis), the measurement of $B_d^0(t) \to \pi^+\pi^-$ can be used to search for new physics in the $b \to d$ FCNC.

Not surprisingly, however, there are some potential problems which must be taken into consideration. Most importantly, there are discrete ambiguities in extracting some of the phases necessary for the analysis. Their presence may make the discovery of new physics difficult, particularly since there will be errors associated with both the experimental measurements and theoretical predictions. In order to remove discrete ambiguities, it is necessary to be able to measure the same quantities in a variety of ways. For example, the search for new physics will be facilitated if we have independent information about α *and* are able to perform an isospin analysis. However, it may happen that, due to small branching ratios or poor detection efficiencies, one cannot measure the rates for the decays $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ and/or $B^+ \to \pi^+\pi^0$. Instead, only upper limits can be obtained, so that a full isospin analysis cannot be performed. In this case, one has to examine the extent to which partial knowledge of these quantities helps in detecting the presence of new physics.

In this paper, we discuss all of these issues. We begin in Sec. 2 with a review of the $B \to \pi\pi$ isospin analysis. Here we show how new physics affects the extraction of $|P/T|$, and present the SM expectations for the magnitude of this ratio. We also discuss the potential difficulties (discrete ambiguities, incomplete isospin analysis) in looking for new physics by combining the isospin analysis with a theoretical prediction for $|P/T|$. In the following two sections, we take $|P/T|$ to lie within a particular range of values, and examine the prospects

for detecting the presence of new physics in $B_d^0(t) \to \pi^+\pi^-$. In Sec. 3 it is assumed that only $B_d^0(t) \to \pi^+\pi^-$ has been measured. Here we also require independent information about 2α. In this scenario, it is possible to detect the presence of new physics, but there are complications due to discrete ambiguities. The prospects for detecting new physics can be significantly improved with the help of other, independent measurements which can be used to remove these discrete ambiguities. In Sec. 4 we examine the effect of combining the measurement of $B_d^0(t) \to \pi^+\pi^-$ with an isospin analysis. Surprisingly, even if no information is available regarding $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ decays, the isospin symmetry nevertheless reduces the discrete ambiguities found in Sec. 3 by a factor of two. Of course, the situation is improved if we do have information about $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$. This can take one of two forms. Either a full isospin analysis is possible, which involves measuring the decays $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$, or we have only limits on the quantities involved in these decays. In either case, one can indeed detect the presence of new physics, but once again discrete ambiguities complicate matters. The situation can be greatly improved if one assumes, as is likely to be the case in practice, that independent information about 2α is available. We conclude in Sec. 5.

2 Theoretical Framework

2.1 Isospin Analysis

We begin with a review of the $B \to \pi\pi$ isospin analysis. In the SM, in the Wolfenstein parametrization, the weak phase of the B_d^0–$\overline{B_d^0}$ mixing amplitude is $e^{-2i\beta}$. When considering B decays, it is useful to remove this mixing phase by redefining the decay amplitudes as follows:

$$A^f \equiv e^{i\beta} Amp(B_d^0 \to f) \quad , \quad \bar{A}^f \equiv e^{-i\beta} Amp(\bar{B}_d^0 \to f) \ . \tag{2}$$

Then the time-dependent decay rate for a $B_d^0(t)$ to decay into a final state f takes the form

$$\Gamma(B_d^0(t) \to f) = e^{-\Gamma t} \left[\frac{|A^f|^2 + |\bar{A}^f|^2}{2} + \frac{|A^f|^2 - |\bar{A}^f|^2}{2} \cos(\Delta M t) - \mathrm{Im} \left(A^{f*} \bar{A}^f \right) \sin(\Delta M t) \right] , \tag{3}$$

where $B_d^0(t)$ is a B-meson which at $t = 0$ was a B_d^0.

In general, the decay $B_d^0 \to \pi\pi$ receives contributions from a tree-level amplitude and a $b \to d$ penguin amplitude. Using the unitarity of the CKM matrix to eliminate the $V_{cb}^* V_{cd}$ piece

of the penguin diagram, we can write

$$A(B_d^0 \to \pi^+\pi^-) \equiv A^{+-} = \sqrt{2}\left[Te^{i\delta}e^{-i\alpha} + Pe^{i\delta_P}e^{-i\theta_{NP}}\right], \qquad (4)$$

where the $Te^{i\delta}$ term includes the u-quark piece of the penguin amplitude, and $Pe^{i\delta_P}$ contains the remaining contributions to the penguin amplitudes. The δ's are strong phases and the electroweak penguin contribution has been ignored [15]. In Eq. (4), we have allowed for the possibility of new physics affecting the $b \to d$ FCNC by including the new-physics phase θ_{NP}. This phase will be nonzero if the B_d^0–$\overline{B_d^0}$ mixing amplitude and the $b \to d$ penguin amplitude are affected by the new physics in different ways. (Note that it is also possible for new physics to affect the magnitudes of T and P. This possibility is implicitly included in our method.) The corresponding $\bar{A}^{+-}$ amplitude is obtained from the A^{+-} amplitude in Eq. (4) by changing the signs of the weak phases α and θ_{NP}.

If the penguin contributions to $B_d^0 \to \pi^+\pi^-$ are negligible, then the measurement of the time-dependent rate for this decay allows one to obtain the CP angle α. From Eq. (3), the coefficient of the $\sin(\Delta M t)$ term probes the relative phase of the A^{+-} and $\bar{A}^{+-}$ amplitudes. And, from Eq. 4 we see that this relative phase is 2α if $P \sim 0$. On the other hand, if P is not negligible, then α cannot be cleanly extracted from this measurement, since the relative phase of A^{+-} and $\bar{A}^{+-}$ is then a complicated function of α and the other parameters.

Under such circumstances, an isospin analysis can be used to cleanly extract α. The amplitude for $B_d^0 \to \pi^+\pi^-$ is related by isospin to the amplitudes for $B_d^0 \to \pi^0\pi^0$ (A^{00}) and $B^+ \to \pi^+\pi^0$ (A^{+0}):

$$A^{+0} = \frac{1}{\sqrt{2}} A^{+-} + A^{00} . \qquad (5)$$

Thus, if we write

$$\begin{aligned} A^{00} &= T^{00}e^{i\delta^{00}}e^{-i\alpha} + P^{00}e^{i\delta_P^{00}}e^{-i\theta_{NP}} , &(6)\\ A^{+0} &= T^{+0}e^{i\delta^{+0}}e^{-i\alpha} , &(7)\end{aligned}$$

the isospin relation [Eq. (5)] implies

$$\begin{aligned} T^{+0}e^{i\delta^{+0}} &= Te^{i\delta} + T^{00}e^{i\delta^{00}}, \\ P^{00}e^{i\delta_P^{00}} &= -Pe^{i\delta_P} . \end{aligned} \qquad (8)$$

The $\bar{A}$ amplitudes obey similar isospin relations.

In order to obtain α, we note that the magnitudes of the six amplitudes $|A^{+-}|$, $|A^{00}|$, $|A^{+0}|$, $|\bar{A}^{+-}|$, $|\bar{A}^{00}|$ and $|\bar{A}^{-0}|$ can be measured experimentally. We can therefore construct the isospin triangles involving the A and $\bar{A}$ amplitudes. Furthermore, as noted above, the relative phase between the A^{+-} and $\bar{A}^{+-}$ amplitudes can be measured in $B_d^0(t) \to \pi^+\pi^-$. This then fixes the relative orientations of the A- and $\bar{A}$-triangles. However, the key point here is that this also fixes the relative orientations of the A^{+0} and $\bar{A}^{-0}$ amplitudes. Since the relative phase of these two amplitudes is just 2α [see Eq. (7)], this shows that the isospin analysis allows one to remove the penguin pollution and cleanly extract α.

Explicitly, α is found as follows. First, we define the relative phase between the A^{+-} and $\bar{A}^{+-}$ amplitudes to be $2\alpha_{eff}^{+-}$. Second, the construction of the A-triangle allows one to measure Φ, the angle between the A^{+0} and A^{+-} amplitude. Similarly, the $\bar{A}$-triangle can be used to obtain $\bar{\Phi}$, the angle between $\bar{A}^{-0}$ and $\bar{A}^{+-}$. Φ and $\bar{\Phi}$ are defined via

$$\begin{aligned} \cos\Phi &= \frac{(\frac{1}{2}|A^{+-}|^2 + |A^{+0}|^2 - |A^{00}|^2)}{\sqrt{2}|A^{+-}||A^{+0}|} ~, \\ \cos\bar{\Phi} &= \frac{(\frac{1}{2}|\bar{A}^{+-}|^2 + |\bar{A}^{-0}|^2 - |\bar{A}^{00}|^2)}{\sqrt{2}|\bar{A}^{+-}||\bar{A}^{-0}|} ~. \end{aligned} \tag{9}$$

Given that 2α is the relative phase between A^{+0} and $\bar{A}^{-0}$, the angle α is then determined by $2\alpha = 2\alpha_{eff}^{+-} + \bar{\Phi} - \Phi$.

Finally, it is useful to examine which measurements are really needed in order to carry out the isospin analysis. This analysis involves six amplitudes: A^{+-}, A^{00}, A^{+0}, $\bar{A}^{+-}$, $\bar{A}^{00}$ and $\bar{A}^{-0}$. Experimentally, at best one can measure the magnitudes and relative phases of these six amplitudes, giving 11 measurements. However, due to the (complex) A and $\bar{A}$ isospin triangle relations, four of the measurements are not independent. Furthermore, we have $|A^{+0}| = |\bar{A}^{-0}|$. Thus, of the 11 measurements, only six are independent. Three of these come from measurements of $B_d^0(t) \to \pi^+\pi^-$:

$$\begin{aligned} B^{+-} &\equiv \frac{1}{2}\left(|A^{+-}|^2 + |\bar{A}^{+-}|^2\right) ~, \\ a_{dir}^{+-} &\equiv \frac{|A^{+-}|^2 - |\bar{A}^{+-}|^2}{|A^{+-}|^2 + |\bar{A}^{+-}|^2} ~, \\ 2\alpha_{eff}^{+-} &\equiv Arg\left(A^{+-*}\bar{A}^{+-}\right) ~. \end{aligned} \tag{10}$$

Two more can be obtained from measurements of $B_d^0 \to \pi^0\pi^0$ and $\overline{B_d^0} \to \pi^0\pi^0$. They are B^{00} and a_{dir}^{00}, defined analogously to the above expressions for B^{+-} and a_{dir}^{+-}. The sixth measurement is taken to be the branching ratio for $B^+ \to \pi^+\pi^0$, $B^{+0} \equiv |A^{+0}|^2$. (Note that, since $|A^{+0}| = |\bar{A}^{-0}|$, B^{+0} is equal to B^{-0}, the branching ratio for $B^- \to \pi^-\pi^0$.) In principle, the measurement of $B_d^0(t) \to \pi^0\pi^0$ would also allow one to measure $\sin(2\alpha_{eff}^{00})$. In practice, however, this is unlikely to be feasible. And in any case, since there are only six independent measurements, $\sin(2\alpha_{eff}^{00})$ can always be expressed in terms of the other measurements. We will thus refer to α_{eff}^{+-} as α_{eff} from now on.

In terms of measurable quantities, the quantities $\cos\Phi$ and $\cos\bar{\Phi}$ defined in Eq. (9) can be expressed as

$$
\begin{aligned}
\cos\Phi &= \frac{\frac{1}{2}B^{+-}(1+a_{dir}^{+-}) + B^{+0} - B^{00}(1+a_{dir}^{00})}{\sqrt{2}\sqrt{B^{+-}(1+a_{dir}^{+-})}\sqrt{B^{+0}}} ~, \\
\cos\bar{\Phi} &= \frac{\frac{1}{2}B^{+-}(1-a_{dir}^{+-}) + B^{+0} - B^{00}(1-a_{dir}^{00})}{\sqrt{2}\sqrt{B^{+-}(1-a_{dir}^{+-})}\sqrt{B^{+0}}} ~. \qquad (11)
\end{aligned}
$$

Note that these quantities depend only on ratios of branching ratios. Thus, the isospin analysis can be carried out with knowledge of only five of the six independent quantities. These are: B^{00}/B^{+-}, B^{+0}/B^{+-}, a_{dir}^{+-}, a_{dir}^{00} and $2\alpha_{eff}$. (Of course, in practice, all six independent measurements will be made.)

2.2 New Physics

The theoretical expressions for the amplitudes [Eqs. (4), (6) and (7)] contain a total of seven physical parameters: α, θ_{NP}, T, T^{00}, P, $\Delta \equiv \delta - \delta_P$ and $\Delta^{00} \equiv \delta^{00} - \delta_P$. With only six experimental measurements, it is obvious that one cannot solve for all these parameters (this was to be expected, given the CKM ambiguity [12]). However, it is useful to express some of these parameters in terms of the measurable quantities and the angles α and θ_{NP}. In particular, we have

$$
P^2 = \frac{B^{+-}}{4\sin^2(\alpha-\theta_{NP})}\left[1 - y\cos(2\alpha - 2\alpha_{eff})\right] ~, \qquad (12)
$$

$$
T^2 = \frac{B^{+-}}{4\sin^2(\alpha-\theta_{NP})}\left[1 - y\cos(2\theta_{NP} - 2\alpha_{eff})\right] ~, \qquad (13)
$$

where we have defined $y \equiv \sqrt{1-(a_{dir}^{+-})^2}$. (Expressions similar to these, for the case $\theta_{NP}=0$, were first derived in Ref. [14].)

The ratio of the magnitudes of the penguin and tree amplitudes for the $B_d^0 \to \pi^+\pi^-$ mode then has the following simple functional form in terms of θ_{NP}:

$$r^2 \equiv \frac{P^2}{T^2} = \frac{1 - y\cos(2\alpha - 2\alpha_{eff})}{1 - y\cos(2\theta_{NP} - 2\alpha_{eff})} ~. \tag{14}$$

From this expression, we can see that, given measurements of α, a_{dir}^{+-} and α_{eff}, and given a theoretical prediction for r, one can obtain θ_{NP}. (Note that a full isospin analysis is not necessary here. If α can be obtained from measurements outside the $B \to \pi\pi$ system, then the measurement of $B_d^0(t) \to \pi^+\pi^-$ is sufficient to obtain r^2.) Obviously, if it is found that $\theta_{NP} \neq 0$, this will indicate the presence of new physics. (Note that if, in reality, $\theta_{NP}=0$ but new physics has affected the magnitudes of P and T, this may still show up as an effective nonzero θ_{NP}. But since we are simply looking for $\theta_{NP} \neq 0$, this distinction is unimportant.)

Of course, theory will not, in general, predict a specific value for r, but rather give a range. And in fact, theoretical estimates of r, assuming no new physics, exist in the literature. Fleischer and Mannel [16] quote the range

$$0.07 \le r \le 0.23 ~. \tag{15}$$

In view of the fact that Ref. [16] does not include the u- and c-quark contributions to the $b \to d$ penguin amplitudes, this range must be expanded. We therefore take what we call the "acceptable range of r" to be

$$0.05 \le r \le 0.5 ~. \tag{16}$$

We should remark here that recent CLEO data [17] finds that the branching ratios for $B \to K\pi$, which are dominated by $b \to s$ penguin amplitudes, are larger than expected. This suggests that the $b \to d$ penguin amplitude may also be larger than expected. In addition, CLEO finds that the branching ratio for $B_d^0 \to \pi^+\pi^-$ is 4×10^{-6}, smaller than expected. Taken together, the data suggest that the P/T ratio may be quite a bit larger than the range shown in Eq. (16), and that P and T interfere destructively to reduce the $B_d^0 \to \pi^+\pi^-$ branching ratio[4].

[4] We note, however, that this naive picture is unlikely to be the full story. This explanation [18] of the measured branching ratios requires $\cos\gamma < 0$, which is disfavoured by the SM [19]. Furthermore, the large branching ratio for $B_d^0 \to K^0\pi^0$ [17] cannot be explained within this picture. It seems likely that more complicated effects, such as final-state interactions or inelastic scattering, are coming into play [20, 21].

Nevertheless, since the point of the paper is to explore the possibilities for finding new physics in $B \to \pi\pi$, we will continue to use the range given in Eq. (16). It may well be that, by the time measurements of $B \to \pi\pi$ decays are done, the theoretical range for r will have changed. However, the techniques described in this paper for finding new physics will still hold, since they do not depend on the exact values chosen for the lower and upper bounds on r.

If, for a certain set of measurements, the value of r obtained assuming $\theta_{NP} = 0$ is outside the range in Eq. (16), this implies that new physics is present. One can then estimate the value of θ_{NP} for which r is lowered to the acceptable range. Of course, this may not be feasible for all possible cases, and one may conclude then that the large r is due to new physics that does not contribute simply to the phase of the penguin diagram, but also alters its magnitude substantially.

2.3 New Physics: Potential Difficulties

In the previous subsection, we showed that, given measurements of α, a_{dir}^{+-} and α_{eff}, along with a theoretical estimate of the ratio of the penguin and tree amplitudes, one can extract θ_{NP}. If θ_{NP} is found to be nonzero, this will establish the presence of new physics. In practice, however, the situation not quite so simple.

First, as noted earlier, there are two ways that information about α can be obtained: either through independent measurements outside the $B \to \pi\pi$ system, or via an isospin analysis. One problem with the isospin method is that it may not be so easy to make all the measurements necessary to carry out the analysis. In particular, the decays $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ may be quite challenging. Given that $B(B_d^0 \to \pi^+\pi^-)$ has been measured to be 4×10^{-6} [17], this suggests that the branching ratio for $B_d^0 \to \pi^0\pi^0$ is even smaller, perhaps considerably so. In addition, the efficiency for the detection of the two π^0 mesons in the final state may not be very high. It is therefore conceivable that it will not be possible to carry out a full isospin analysis, at least at first-generation B-factories. (On the other hand, if the $b \to d$ penguin is indeed large, as suggested by the latest CLEO data [17], then $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ may well be dominated by its penguin contributions, leading to a large branching ratio. Indeed, recent analyses [21] of the CLEO data, which includes inelastic rescattering effects, predict $B(B_d^0 \to \pi^0\pi^0) \sim 5 \times 10^{-6}$. This branching ratio is about an order of magnitude larger than earlier estimates based on

factorization. Thus, at this point in time, it is not clear how difficult it will be to perform an isospin analysis of $B \to \pi\pi$ decays.)

Second, there are serious complications due to discrete ambiguities, and this holds regardless of whether or not an isospin analysis is done. Suppose, first, that one can perform the isospin analysis. In this case, using isospin relations, the angle α is determined by $2\alpha = 2\alpha_{eff} + \bar{\Phi} - \Phi$, where Φ and $\bar{\Phi}$ are obtained from Eq. (9). Since only $\cos\Phi$ and $\cos\bar{\Phi}$ are known, there is a twofold ambiguity in each of Φ and $\bar{\Phi}$, i.e. $\pm\Phi$ as well as $\pm\bar{\Phi}$ are allowed in the equation for α. In addition, since it is the quantity $\sin 2\,\alpha_{eff}$ which is measured, $2\alpha_{eff}$ is also determined up to a twofold ambiguity. Hence, 2α is obtained with an eightfold ambiguity.

The ratio r^2 itself has a fourfold ambiguity [see Eq. (14)]: the quantity $\cos(2\alpha - 2\alpha_{eff})$ takes two values, as does $2\alpha_{eff}$. In general, then, we will find four distinct possible values of r^2 for the same set of observables. This may make it difficult to determine if new physics is present: if only one of the four values of r^2 at $\theta_{NP} = 0$ lies within the acceptable range, then the measurements may be consistent with the SM. One cannot unequivocally conclude that there is new physics (though there might be).

If the isospin analysis cannot be performed, then the CP phase 2α cannot be extracted cleanly from measurements of $B_d^0(t) \to \pi^+\pi^-$. In such a case, in order to use Eq. (14), we will need to obtain knowledge of 2α from other measurements[5]. This can be done in several ways. For example, if the CP angles β and γ are extracted via $B_d^0(t) \to J/\Psi K_S$ and $B^\pm \to DK^\pm$, respectively, this will give us information about 2α due to the unitarity triangle condition $2\alpha + 2\beta + 2\gamma = 0 \pmod{2\pi}$. However, as will be explained in the next section, this only determines 2α up to a fourfold ambiguity, which, along with the twofold ambiguity in $2\alpha_{eff}$, still leaves an eightfold ambiguity in r^2. A more promising source of information is a $B \to \rho\pi$ Dalitz plot analysis [22]. With this method, one can obtain 2α with no discrete ambiguity. In this case, one is left with a twofold ambiguity in r^2.

Ideally, we will be able to perform a complete isospin $B \to \pi\pi$ analysis *and* have unambiguous knowledge of 2α from $B \to \rho\pi$. In this case, we will obtain a single value of r^2, which

[5]Note that the independent knowledge of α does not resolve the CKM ambiguity [12]. The determination of α in the isospin analysis decouples from the solutions of the other parameters, and hence θ_{NP} still cannot be determined without fixing one of the theoretical parameters.

will allow us to test unambiguously for the presence of new physics.

Unfortunately, in the real world we will probably have to deal with one of the scenarios which gives r^2 with some number of discrete ambiguities. In the next two sections, we will analyze all of these scenarios. As we will see, even despite the presence of discrete ambiguities, and even if a full isospin analysis cannot be performed, there is still a significant region of parameter space where the presence of new physics can be clearly established.

3 Only $B_d^0(t) \to \pi^+\pi^-$ is Measured

We first suppose that only $B_d^0(t) \to \pi^+\pi^-$ has been measured. In this case, we will not have clean knowledge of the CP phase α. In order to use Eq. (14) to search for new physics, it will then be necessary to obtain knowledge of α from independent measurements. One possibility is to use the fact that, even in the presence of new physics, the three angles α, β and γ still correspond to the interior angles of a triangle [23]. That is, we have $2\alpha+2\beta+2\gamma = 0 \pmod{2\pi}$. Thus, measurements of 2β and 2γ will indirectly give us information about 2α, even if new physics is present.

When one probes the CP phase β via $B_d^0(t) \to J/\Psi K_S$, the function one extracts is $\sin 2\beta$. This then determines 2β up to a twofold ambiguity. Similarly, the measurement of CP violation in $B^\pm \to DK^\pm$ gives $\sin^2\gamma$ (or equivalently $\cos 2\gamma$) which also yields 2γ up to a twofold ambiguity. Using the triangle condition, these two measurements therefore determine 2α with a fourfold ambiguity. Since the measurement of $B_d^0(t) \to \pi^+\pi^-$ allows one to extract $\sin 2\alpha_{eff}$, which determines $2\alpha_{eff}$ up to a twofold discrete ambiguity, in total there is an eightfold ambiguity in the determination of r^2. With such a large number of possible r^2 solutions, it is very likely that at least one of them will lie within the acceptable r^2 region [Eq. (16)], in which case one cannot be sure that new physics is present.

The situation can be improved in a variety of ways. There are methods which use indirect, mixing-induced CP violation to extract functions of β and γ other than $\sin 2\beta$ and $\sin 2\gamma$. This additional knowledge will remove the discrete ambiguity in 2β and/or 2γ. For example, a Dalitz-plot analysis of the decay $B_d^0(t) \to D^+D^-K_S$ allows one to extract the function $\cos 2\beta$ [24]. This function can also be obtained through a study of $B_d^0 \to \Psi + K \to$

$\Psi + (\pi^- \ell^+ \nu)$, known as "cascade mixing" [25]. Knowledge of both $\sin 2\beta$ and $\cos 2\beta$ determines 2β without ambiguity. Similarly, $\sin 2\gamma$ can be obtained from $B_s^0(t) \to D_s^\pm K^\mp$ if the width difference between the two B_s mass eigenstates is measurable [26], and this additional knowledge removes the ambiguity in 2γ. Finally, a Dalitz-plot analysis of $B_d^0(t) \to D^\pm \pi^\mp K_S$ can be used to obtain the phase $2(2\beta + \gamma)$ without ambiguity [24], and this knowledge will reduce the discrete ambiguity in both 2β and 2γ. Depending on which of these measurements are made, the discrete ambiguity in r^2 can be reduced to a fourfold or even a twofold ambiguity.

It is also possible to get at 2α directly. If one performs a Dalitz-plot analysis of $B \to \rho\pi$ decays, both $\sin 2\alpha$ and $\cos 2\alpha$ can be extracted [22]. This then determines 2α with no ambiguity. In this case, one is left with a twofold discrete ambiguity in r^2, due entirely to the discrete ambiguity in $2\alpha_{eff}$.

For all possible scenarios of this type, the prospects for discovering new physics can be summarized in Fig. 1. We consider 12 specific values of 2α, varying between 0 and 2π. For a given value of 2α, we show the region in $2\alpha_{eff}$–a_{dir}^{+-} space which is consistent with the SM. That is, the region contains those values of $2\alpha_{eff}$ and a_{dir}^{+-} for which the ratio r satisfies the bound of Eq. (16). Note that, for a given value of 2α, there are two allowed $2\alpha_{eff}$–a_{dir}^{+-} regions. One of these regions is for $2\alpha_{eff}$, while the other corresponds to $\pi - 2\alpha_{eff}$, which reflects the fact that $2\alpha_{eff}$ can only be measured up to a twofold ambiguity.

Depending on which measurements have been made, r^2 will be determined with an N-fold ambiguity ($N = 2, 4, 8$). In a particular scenario, in order to see whether the measurements indicate the presence of new physics, one has to consider the N values of the pair $(2\alpha, 2\alpha_{eff})$. If (at least) one of these sets of values corresponds to a point in the appropriate plot which is consistent with the SM, then one cannot conclude that new physics is present. However, if all such values correspond to points in the plots which lie outside the SM-allowed regions, then this is a clear signal of new physics.

To give an example of how this works, suppose that the Dalitz-plot $B \to \rho\pi$ analysis is performed, and it is found that $2\alpha = 180°$ (present data suggests that $\alpha \simeq 90°$ is the preferred SM value [19]). If the measurement of $B_d^0(t) \to \pi^+\pi^-$ yields $\sin 2\alpha_{eff} = 0.966$ (i.e. $2\alpha_{eff} = 75°$ or $105°$), then, regardless of the value of a_{dir}^{+-}, this indicates the presence of new physics. On the other hand, if one finds $\sin 2\alpha_{eff} = 0$, then new physics is implied only if $|a_{dir}^{+-}| \gtrsim 0.8$.

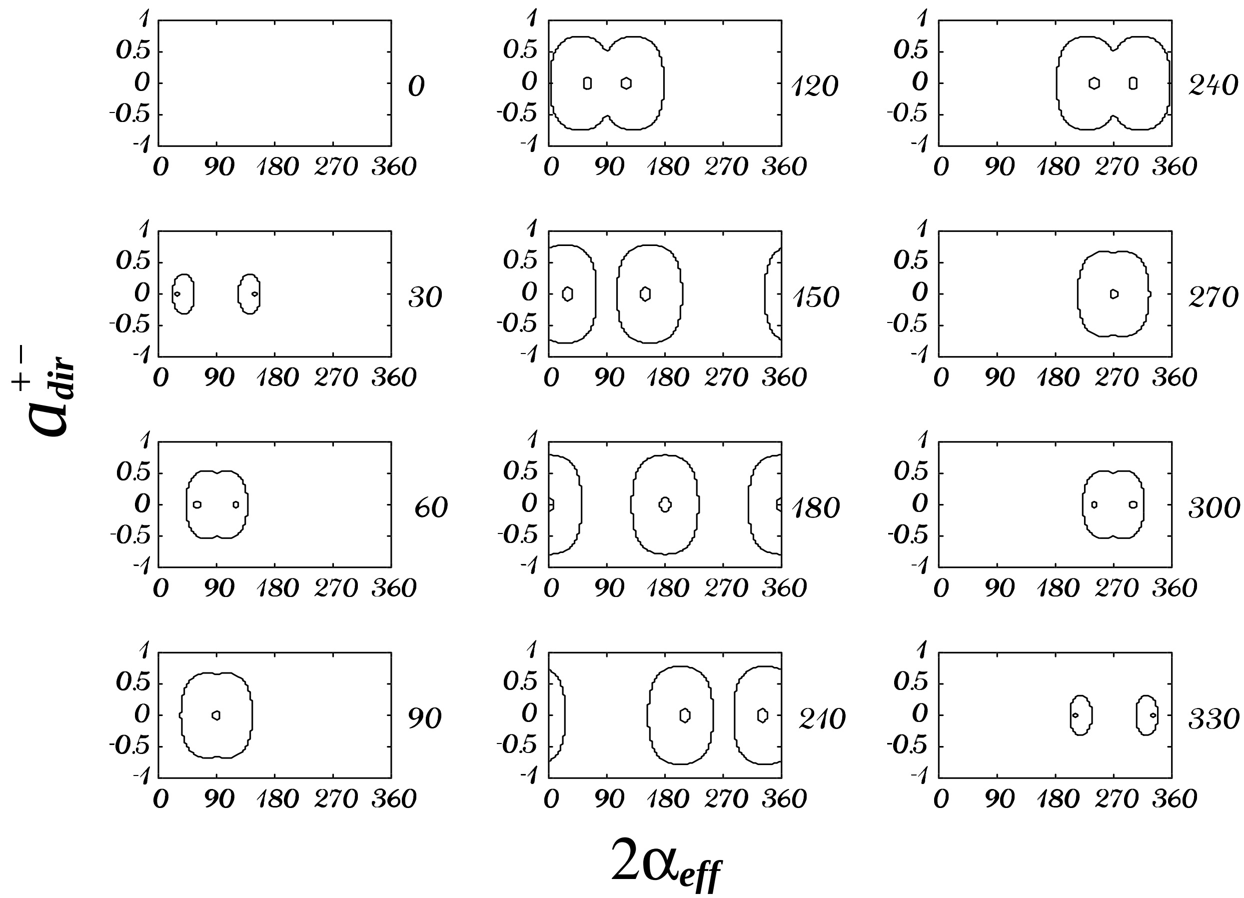

Figure 1: The region in $2\alpha_{eff}$–a^{+-}_{dir} space which is consistent with the theoretical prediction for $|P/T|$ [Eqs. (14),(16)], for various values of 2α. It is assumed that only $B^0_d(t) \to \pi^+\pi^-$ has been measured. In all figures, the x-axis is $2\alpha_{eff}$ and the y-axis is a^{+-}_{dir}. The value of 2α used in a particular figure is given to the right of that figure.

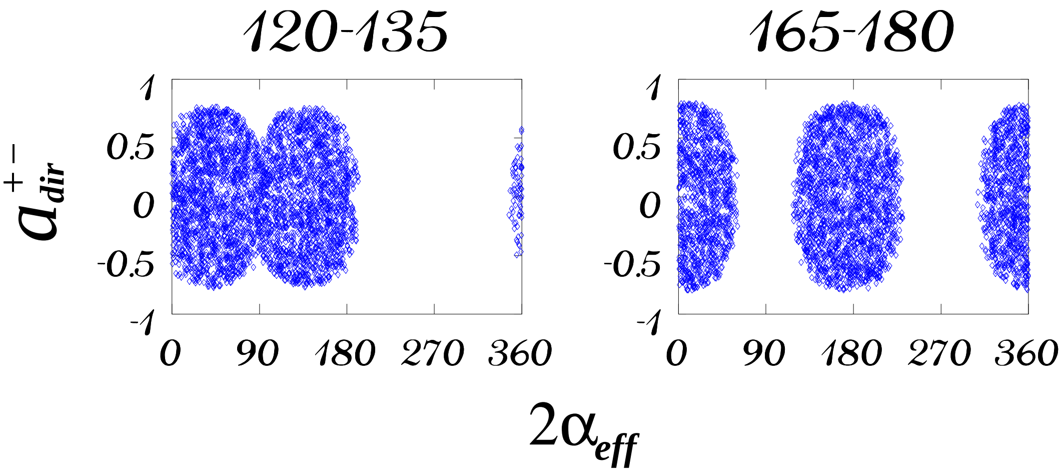

Figure 2: As in Fig. 1, except that 2α is allowed to take a range of values. The range of values of 2α used in a particular figure is given above that figure.

We therefore see that the measurement of $B_d^0(t) \to \pi^+\pi^-$, when combined with independent knowledge of 2α, can reveal the presence of new physics, given a reliable prediction of the ratio P/T in the SM.

However, discrete ambiguities can muddy the picture considerably. For example, if r^2 is only known up to an 8-fold ambiguity, then one must essentially superimpose 4 plots of the type shown in Fig. 1, in which case there are very few values of the pair $(2\alpha_{eff}, a_{dir}^{+-})$ which point unequivocally to new physics. For this reason it is important to be able to reduce the discrete ambiguity in r^2 as much as possible.

This point is made even sharper when one considers the fact that all measurements will include experimental errors. In Fig. 2, we assume that 2α is known to be within a certain range ($120^\circ \le 2\alpha \le 135^\circ$ [left-hand figure of Fig. 2] or $165^\circ \le 2\alpha \le 180^\circ$ [right-hand figure of Fig. 2]). We then show the region in $2\alpha_{eff}$–a_{dir}^{+-} space which is consistent with the SM. In this case the allowed region is visibly larger than that presented in the plots of Fig. 1. It is therefore clear that if, due to the discrete ambiguity in r^2, one is forced to superimpose several such figures, the prospects for detecting new physics will be considerably reduced.

Fortunately, the above analysis does not tell the whole story. Indeed, this analysis is incomplete: it does not take into account the fact that the isospin analysis must reproduce the independently-measured value of 2α. Now, it is rather obvious that, if the decays $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ can be measured, and an isospin analysis performed, this additional

constraint will reduce the $2\alpha_{eff}$–a_{dir}^{+-} region which is consistent with the SM. However it is also true that even if we have *no* information about $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$, the fact that one must be able to reproduce 2α using isospin is sufficient to remove the twofold discrete ambiguity in $2\alpha_{eff}$ which appears in all the plots of Figs. 1 and 2! This remarkable result is discussed in the next section, along with an examination of how actual measurements of, or limits on, $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ can improve the prospects for the detection of new physics.

4 Beyond $B_d^0(t) \to \pi^+\pi^-$

In practice, it is likely that we will have more information about $B \to \pi\pi$ decays than just the measurement of $B_d^0(t) \to \pi^+\pi^-$. In the most optimistic scenario, the decays $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ will both be measured, which will allow us to obtain the quantities B^{+0}, B^{00} and a_{dir}^{00}. In this case the full isospin analysis can be carried out, so that 2α can be determined. With this knowledge, one can then use r^2 [Eq. (14)] to search for new physics. (Note that, as discussed in Sec. 2.1, in fact only the ratios of branching ratios B^{+0}/B^{+-} and B^{00}/B^{+-} are needed to perform the isospin analysis.)

However, there are problems with this procedure. First, there will be errors associated with all measured quantities, which will lead to a range of allowed values for 2α. Second, as discussed in Sec. 2.3, the isospin analysis only determines 2α and r^2 up to an eightfold and fourfold ambiguity, respectively. As we saw in the previous section, these two facts may make it difficult to definitively establish the presence of new physics.

This situation can be improved if, in addition to the $B \to \pi\pi$ analysis, we have information about 2α from other measurements. In fact, this is quite likely: by the time $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ are measured, experiments which yield independent information about 2α will probably have been performed. Various possibilities have been discussed in the previous section. For example, the measurements of $\sin 2\beta$ and $\cos 2\gamma$, combined with the triangle relation $2\alpha + 2\beta + 2\gamma = 0 \pmod{2\pi}$, determine 2α up to a fourfold ambiguity. But this discrete ambiguity can be reduced by comparing these four solutions with the eight obtained from the isospin analysis. It is straightforward to see that, in general, there are only two val-

ues of 2α which are common to the four solutions here and the eight solutions found in the isospin analysis. In addition, for a given value of 2α, the value of $2\alpha_{eff}$ is fixed. *That is, the discrete ambiguity in* $2\alpha_{eff}$ *which affected the analysis of Sec. 3 has been removed here.* The two solutions are then

$$(2\alpha, 2\alpha_{eff}) \ , \quad (\pi - 2\alpha, \pi - 2\alpha_{eff}) \ , \tag{17}$$

and lead to a twofold ambiguity in r^2. Furthermore, if both $\sin 2\alpha$ and $\cos 2\alpha$ can be measured via a Dalitz-plot analysis of $B \to \rho\pi$ decays [22], the remaining twofold ambiguity will be lifted. In this case only the true $(2\alpha, 2\alpha_{eff})$ solution will remain, corresponding to a single value of r^2. This is the key point: given an independently-determined value of 2α, the isospin analysis removes the twofold discrete ambiguity in $2\alpha_{eff}$, and hence in r^2. As we will see below, this is an important ingredient in searching for new physics.

Thus, by combining an isospin analysis with independent knowledge of 2α, one can reduce the discrete ambiguity in r^2, thereby improving the prospects for discovering new physics. In this section, we assume that such independent knowledge of 2α will in fact be available.

The prescription to search for new physics then proceeds as follows. For a given set of B^{+0}/B^{+-}, B^{00}/B^{+-} and a_{dir}^{00} measurements, we can calculate which values of a_{dir}^{+-} and $2\alpha_{eff}$ produce values of α which lie within the measured range. One can then check further to see which of these values of $2\alpha_{eff}$ and a_{dir}^{+-} also give r^2 within the allowed theoretical range [Eq. (16)]. If the measured values of a_{dir}^{+-} and $2\alpha_{eff}$ do not satisfy these two conditions, then this is evidence for new physics.

Of course, in practice the measurements of B^{+0}/B^{+-}, B^{00}/B^{+-} and a_{dir}^{00} will have errors associated with them. In this case one can still use the above procedure, except that one must scan over the allowed ranges for these quantities.

In the above discussion, we have assumed that the quantities B^{+0}/B^{+-}, B^{00}/B^{+-} and a_{dir}^{00} have been actually measured. However this may not turn out to be the case: since $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ may be difficult to measure, we may only have limits on these quantities. Fortunately, assuming that independent information about 2α will be available, the above prescription can be carried out even in this scenario. All that changes is that the allowed ranges for B^{+0}/B^{+-}, B^{00}/B^{+-} and a_{dir}^{00} are (presumably) larger than in the case where they

	a^{00}_{dir}	B^{00}/B^{+-}	B^{+0}/B^{+-}
Case A	$-1 - 1$	any value	any value
Case B	$-1 - 1$	$0 - 0.1$	$0.8 - 0.9$
Case C	$0.5 - 0.7$	$0.7 - 0.8$	$0 - 0.5$
Case D	$0.6 - 1$	$0.2 - 0.4$	$0.6 - 0.7$
Case E	$0.6 - 1$	$0.2 - 0.4$	$0.2 - 0.3$

Table 1: The assumed ranges for a^{00}_{dir}, B^{00}/B^{+-} and B^{+0}/B^{+-} for five (hypothetical) sets of experimental measurements.

are measured (with errors).

To illustrate how this all works, we consider a variety of hypothetical experimental measurements. First, we take 2α, assumed to have been obtained from measurements outside the $B \to \pi\pi$ system, to lie within a given domain. (This corresponds roughly to including an experimental error.) We consider two such domains: (i) $165^\circ \le 2\alpha \le 180^\circ$ and (ii) $120^\circ \le 2\alpha \le 135^\circ$.

Second, we assume that a^{00}_{dir}, B^{00}/B^{+-} and B^{+0}/B^{+-} each lie in a specified range. For these allowed ranges, we consider five distinct cases, shown in Table 1. In Case A, it is assumed that $B^0_d/\overline{B^0_d} \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ have not been measured at all, so that we have no knowledge of a^{00}_{dir}, B^{00}/B^{+-} and B^{+0}/B^{+-}. In Case B, the assumptions are that (i) $B^0_d/\overline{B^0_d} \to \pi^0\pi^0$ is not well-measured, so that we have no knowledge of a^{00}_{dir}, and only an upper limit on B^{00}/B^{+-}, and (ii) we have good knowledge of B^{+0}/B^{+-} from the measurement of $B^+ \to \pi^+\pi^0$. In Case C, it is assumed that (i) $B^0_d/\overline{B^0_d} \to \pi^0\pi^0$ is well-measured, so that we have rather precise knowledge of a^{00}_{dir} and B^{00}/B^{+-}, but (ii) we have only an upper limit on B^{+0}/B^{+-}. Finally, in Cases D and E, all quantities are assumed to be known; only the range for B^{+0}/B^{+-} differs between the two cases.

We are now in a position to apply the above prescription to search for new physics. For a given range of 2α, and for a given case, we use a random number generator to obtain values of a^{00}_{dir}, B^{00}/B^{+-} and B^{+0}/B^{+-} in the specified range, and a^{+-}_{dir} and $2\alpha_{eff}$ in the full allowed range. We generate 10^5 sets of values for these five parameters. For a given set, we then check to see whether these parameters produce values for 2α and r^2 which lie within their allowed ranges. In this way we map out the region of $2\alpha_{eff}$–a^{+-}_{dir} space which is consistent with the SM.

The results are shown in Fig. 3. In all cases, by comparing the SM-allowed $2\alpha_{eff}$–a_{dir}^{+-} region with that shown in Fig. 2, one can see the extent to which the prospects for detecting new physics are improved through considerations of isospin.

Consider first Case A. Here, as was the case in Sec. 3, it is assumed that we have no knowledge at all of a_{dir}^{00}, B^{00}/B^{+-} and B^{+0}/B^{+-}. However, the difference here is that, despite this lack of knowledge, we nevertheless require that an isospin analysis yield a value of 2α which lies in the assumed range. By comparing Figs. 2 and 3 for this case, one sees that this condition is sufficient to remove one of the two solutions in Fig. 2. In other words, for that solution, there are no values of a_{dir}^{00}, B^{00}/B^{+-} and B^{+0}/B^{+-} which will simultaneously give 2α and r^2 in their respective allowed ranges. The removal of one solution will always occur as long as the experimental range of 2α is sufficiently restricted so as not to include both 2α and $\pi - 2\alpha$ values. This demonstrates the power of the isospin analysis: even if $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ cannot be measured, the isospin symmetry is able to remove the discrete ambiguity in $2\alpha_{eff}$ which appears in all the plots of Figs. 1 and 2.

We now turn to Case B, in which it is assumed that the ranges for the branching ratios B^{00}/B^{+-} and B^{+0}/B^{+-} are reasonably well known (though there is still only an upper limit on B^{00}/B^{+-}). In this case, even though we still have no knowledge of a_{dir}^{00}, Fig. 3 shows that there is nevertheless a marked reduction in the allowed $2\alpha_{eff}$–a_{dir}^{+-} region. Compared to the case where only $B_d^0(t) \to \pi^+\pi^-$ has been measured (Case A), the $2\alpha_{eff}$–a_{dir}^{+-} region consistent with the SM has been reduced by about a factor of two.

One can do even better if all of the three quantities a_{dir}^{00}, B^{00}/B^{+-} and B^{+0}/B^{+-} are measured reasonably well. Depending on the measured values of these quantities, the allowed $2\alpha_{eff}$–a_{dir}^{+-} region can be reduced even further, as the plots for Cases C, D and E show.

Of course, in order to compute the full allowed $2\alpha_{eff}$–a_{dir}^{+-} region, one will have to superimpose a certain number of plots of this type, depending on the size of the discrete ambiguity in r^2. As always, if the measured values of $2\alpha_{eff}$ and a_{dir}^{+-} lie outside the allowed region, then this will indicate the presence of new physics.

Obviously, it is very unlikely that any of the hypothetical cases considered above will turn out to be the actual experimental situation. Indeed, even the theoretical situation — namely

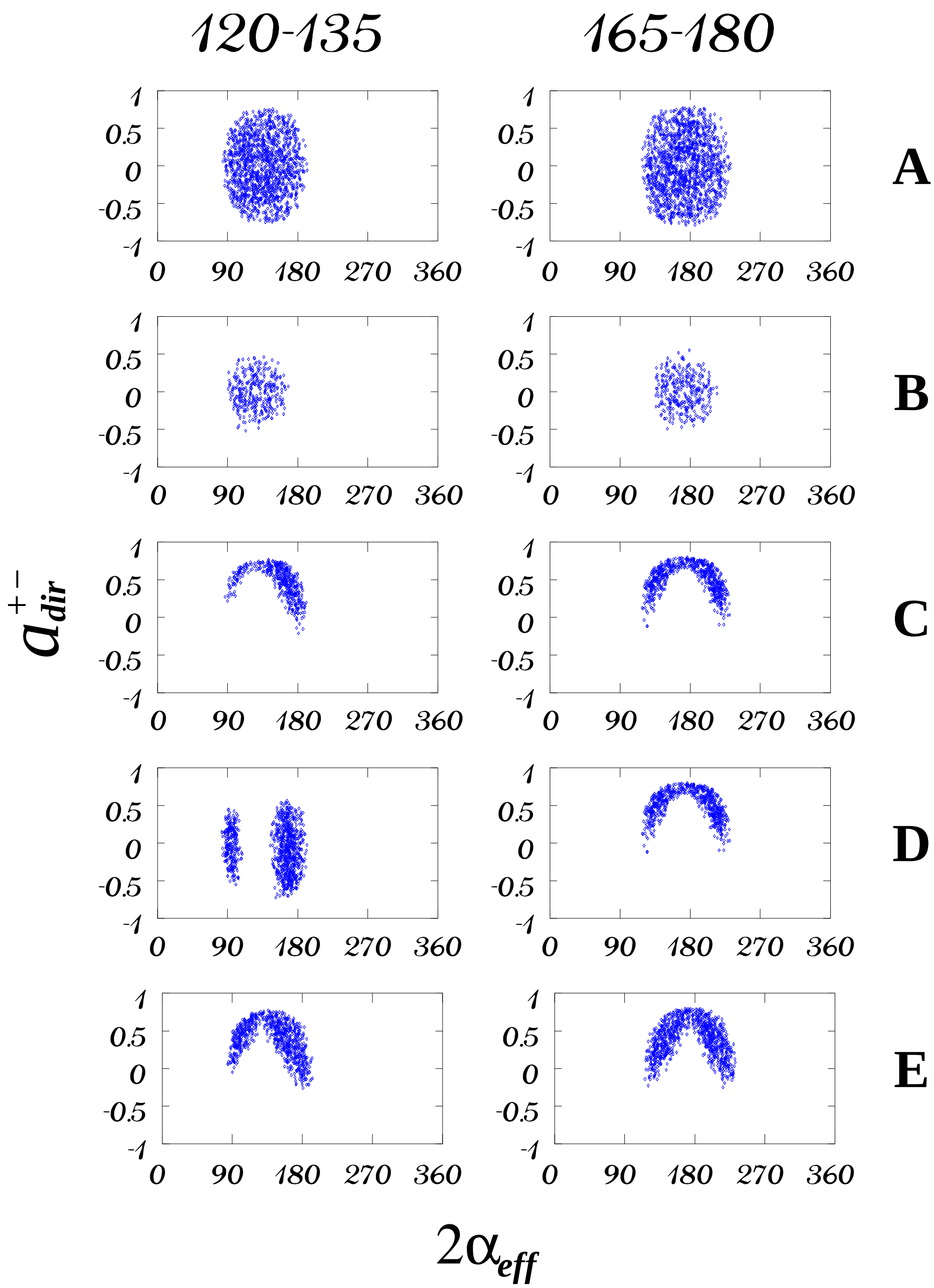

Figure 3: The region in $2\alpha_{eff}$–a_{dir}^{+-} space which is consistent with the theoretical prediction for $|P/T|$ [Eqs. (14),(16)]. In addition to the measurement of $B_d^0(t) \to \pi^+\pi^-$, it is assumed that information about $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ is available. For this latter information, the five scenarios of Table 1 are considered from top (Case A) to bottom (Case E). In all cases, 2α is allowed to take a range of values, given above each of the two columns of figures. In all figures, the x-axis is $2\alpha_{eff}$ and the y-axis is a_{dir}^{+-}.

the allowed range for r^2 — may change by the time the measurements are done. However, regardless of the experimental and theoretical numbers, the analysis described here can be used to search for new physics.

5 Conclusions

In the near future, measurements will be made which will permit us to extract the CKM angles α, β and γ from CP-violating rate asymmetries in the B system. Hopefully, these measurements will reveal the presence of new physics.

There are a variety of methods to test for new physics in the $b \to s$ flavour-changing neutral current (FCNC). However, there is no way to *cleanly* detect new physics in the $b \to d$ FCNC. In order to search for new physics in $b \to d$ transitions, one always needs some theoretical input. That is, one needs to make an assumption regarding hadronic parameters.

We have applied this idea to $B \to \pi\pi$ decays. If the decay $B_d^0(t) \to \pi^+\pi^-$ were dominated by a tree-level amplitude (T), then the angle α could be obtained with no hadronic uncertainties. Unfortunately, this decay also receives a contribution from a penguin amplitude (P) which may be sizeable, spoiling the clean extraction of α. However, it is well known that, by also measuring the decays $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$, one can use isospin to remove the penguin pollution and hence obtain a clean measurement of α. In this paper, we have shown that, by making an assumption about the relative size of P and T, this isospin analysis can also be used to test for the presence of new physics.

In fact, it is not even necessary to measure the decays $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$. If independent information about α is available, then, given a prediction for the allowed range of $|P/T|$, the measurement of the decay $B_d^0(t) \to \pi^+\pi^-$ is sufficient to test for the presence of new physics. Here the principle obstacle is the presence of discrete ambiguities. In the simplest scenario, 2α will probably only be known up to a fourfold ambiguity. In this case, the measurement of $B_d^0(t) \to \pi^+\pi^-$ yields eight possible values for $|P/T|$. By performing an isospin analysis (which can be done even though no information from $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ decays is available!), one can reduce this to four possible $|P/T|$ values. Still, even if there is new physics, it is quite likely that one of these values will lie in the range for $|P/T|$ allowed by

the standard model, thereby masking the presence of new physics. Thus, in order to search for new physics using only $B_d^0(t) \to \pi^+\pi^-$, it will be important to make additional measurements to reduce the discrete ambiguity in 2α.

If it is possible to perform a full isospin analysis, then independent knowledge of α is not needed – the isospin analysis itself yields 2α. However, here too the presence of discrete ambiguities may make it difficult to say with certainty that new physics is present. On the other hand, by the time the full isospin analysis is done, it is quite likely that we *will* have independent information about α. By combining this information with that obtained from the isospin analysis, one can reduce the discrete ambiguities substantially, thereby greatly improving the prospects for detecting new physics.

In summary, we see that there are a variety of scenarios to consider, depending on which measurements have been done. However, in all cases, the bottom line is the following: the analysis of $B \to \pi\pi$ decays, combined with a theoretical prediction for the allowed range of $|P/T|$, can be used to search for the presence of new physics in the $b \to d$ FCNC.

Acknowledgments

N.S. and R.S. thank D.L. for the hospitality of the Université de Montréal, where this work was initiated. The work of D.L. was financially supported by NSERC of Canada.

References

[1] C. Caso et al. (Particle Data Group), *Eur. Phys. J.* **C3** (1998) 1.

[2] For a review, see, for example, *The BaBar Physics Book*, eds. P.F. Harrison and H.R. Quinn, SLAC Report 504, October 1998.

[3] M. Gronau and D. Wyler, *Phys. Lett.* **265B** (1991) 172. See also M. Gronau and D. London, *Phys. Lett.* **253B** (1991) 483; I. Dunietz, *Phys. Lett.* **270B** (1991) 75. Improvements to this method have been discussed by D. Atwood, I. Dunietz and A. Soni, *Phys. Rev. Lett.*

78 (1997) 3257, and have been used in N. Sinha and R. Sinha, *Phys. Rev. Lett.* **80** (1998) 3706.

[4] D. London and R. Peccei, *Phys. Lett.* **223B** (1989) 257; M. Gronau, *Phys. Rev. Lett.* **63** (1989) 1451, *Phys. Lett.* **300B** (1993) 163; B. Grinstein, *Phys. Lett.* **229B** (1989) 280.

[5] M. Gronau and D. London, *Phys. Rev. Lett.* **65** (1990) 3381.

[6] C.O. Dib, D. London and Y. Nir, *Int. J. Mod. Phys.* **A6** (1991) 1253.

[7] Y. Grossman and M.P. Worah, *Phys. Lett.* **395B** (1997) 241.

[8] B. Kayser and D. London, *Phys. Rev.* **D61**: 116012 (2000).

[9] R. Aleksan, I. Dunietz and B. Kayser, *Zeit. Phys.* **C54** (1992) 653.

[10] D. London and A. Soni, *Phys. Lett.* **407B** (1997) 61.

[11] L. Wolfenstein, *Phys. Rev. Lett.* **51** (1983) 1945.

[12] D. London, N. Sinha and R. Sinha, *Phys. Rev.* **D60**: 074020 (1999).

[13] A.J. Buras and R. Fleischer, *Phys. Lett.* **341B** (1995) 379.

[14] J. Charles, *Phys. Rev.* **D59** (1999) 054007.

[15] The effects of electroweak penguin amplitudes on the extraction of α via a $B \to \pi\pi$ isospin analysis are expected to be small, see M. Gronau, O. Hernández, D. London and J. Rosner, *Phys. Rev.* **D52** (1995) 6374.

[16] R. Fleischer and T. Mannel, *Phys. Lett.* **397B** (1997) 269.

[17] CLEO Collaboration, CLNS 99/1650, CLEO 99-18, hep-ex/0001010, January, 2000.

[18] N.G. Deshpande, X.G. He, W.S. Hou and S. Pakvasa, *Phys. Rev. Lett.* **82** (1999) 2240; X.G. He, W.S. Hou and K.C. Yang, *Phys. Rev. Lett.* **83** (1999) 1100; W.S. Hou and K.C. Yang, *Phys. Rev.* **D61**: 073014 (2000).

[19] See, for example, A. Ali and D. London, *Eur. Phys. J.* **C9** (1999) 687.

[20] D. Atwood and A. Soni, *Phys. Lett.* **466B** (1999) 326.

[21] W.-S. Hou and K.-C. Yang, *Phys. Rev. Lett.* **84** (2000) 4806; Y.F. Zhou, Y.L. Wu, J.N. Ng and C.Q. Geng, hep-ph/0006225, June 2000.

[22] A.E. Snyder and H.R. Quinn, *Phys. Rev.* **D48** (1993) 2139.

[23] Y. Nir and D. Silverman, *Nucl. Phys.* **B345** (1990) 301.

[24] J. Charles, A. Le Yaouanc, L. Oliver, O. Pène and J.-C. Raynal, *Phys. Lett.* **425B** (1998) 375.

[25] Y. Azimov, *Phys. Rev.* **D42** (1990) 3705; B. Kayser and L. Stodolsky, Max Planck Institute preprint MPI-PHT-96-112, hep-ph/9610522; B. Kayser, *'97 Electroweak Interactions and Unified Theories*, ed. J. Tran Thanh Van (Editions Frontières, Paris 1997), p. 389.

[26] See, for example, I. Dunietz, *Phys. Rev.* **D52** (1995) 3048, and references therein.